RADAR EQUATION

ECE 514E-RADAR & SATELLITE ENGINEERING

Monday, 21 September 2026

WHERE WE ARE IN SYLLABUS?

ECE 514E – RADAR and SATELLITE ENGINEERING (3 Units)

Pre-requisites: ECE 328 - Principles of Communication Systems, ECE 416 - Digital Communication Systems

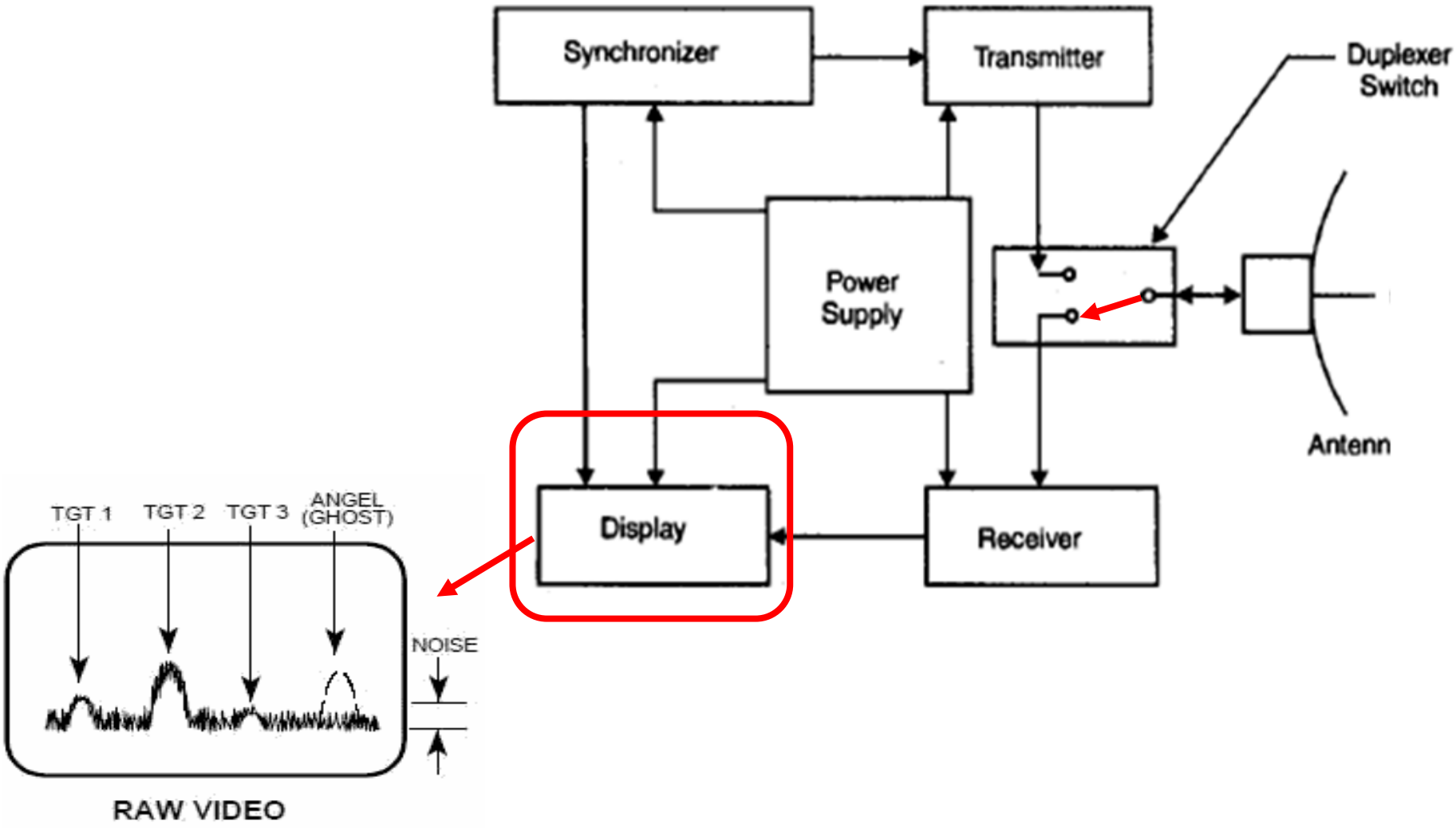
Course Purpose:
To enable students be familiarize with modern radar and navigation systems and principles of design radio-navigation and location systems, as well as radar systems

Expected Learning Outcomes:
Upon completion of this course, a student should be able to:

i. explain principles of operation of radar systems;
ii. apply theoretical and practical knowledge about principles, methods and applications of modern radar systems;
iii. measure and identify the parameters which determine the performance of radar systems;
iv. explain the function of satellite subsystems;
v. apply orbital mechanics formula and tools to spacecraft mission design;
vi. select appropriate launch systems and understand their effect on satellite and payload design and performance;
vii. design and analyse satellite links;
viii. evaluate satellite subsystem performance

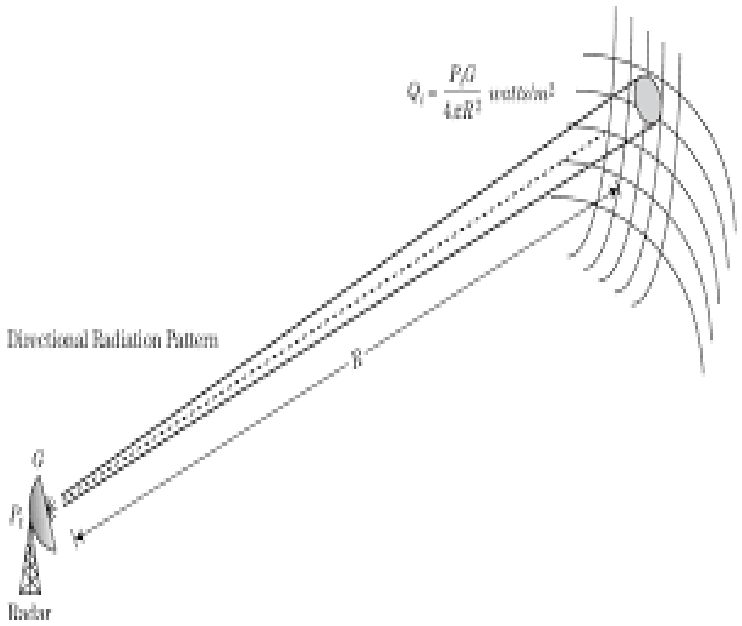
Course Content:
Basic Principles of Radar, Antenna parameters, Radar equation. Performance parameters, target cross-section, MTI and Doppler radar: Doppler Effect, CW radar, FM CW, Delay line cancellers, Pulse Doppler Radar. Scanning, Duplexers and Radar receivers: Sequential lobbing, Conical Scanning, Monopulse Tracking RADAR, Tracking with surveillance RADAR, Acquisition, Radar receiver, Radar Displays; Duplexers.

ASSUMED KNOWLEDGE FROM PREVIOUS CLASS - BLOCK DIAGRAM OF A BASIC MONOSTATIC PULSE RADAR SYSTEM



RADAR EQUATION: BASIC ASSUMPTIONS

1. Electromagnetic waves propagate under ideal conditions, i.e. without dispersion.
2. If high-frequency energy is emitted by an isotropic radiator, then the energy propagate uniformly in all directions.
3. Areas with the same power density therefore form spheres with area, $A = 4\pi R^2$ around the radiating element.



RADAR EQUATION: DEFINITION (1)

1. **Radar equation** represents the physical dependences of the transmit power, that is the wave propagation up to the receiving of the echo-signals.

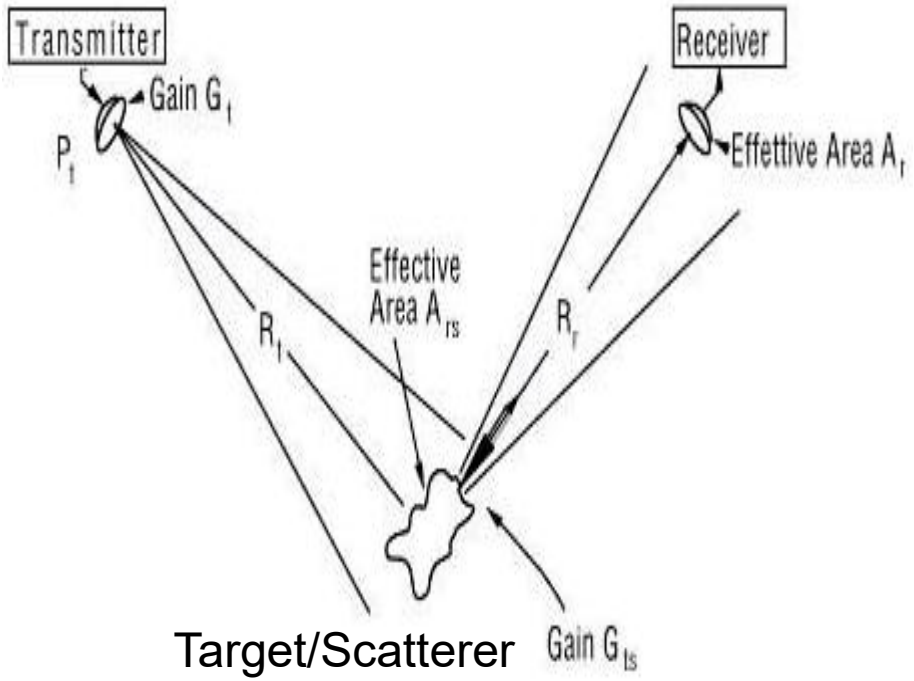
2. **Echo power P_r** returning to the receiving antenna is given by the radar equation, depending on the transmitted power P_t, the slant range **R**, and the reflecting characteristics of the target normally referred to as the the radar scattering cross section).

3. At known **sensitivity of the radar receiver**, the radar equation determines what can be achieved theoretically at a given radar range.

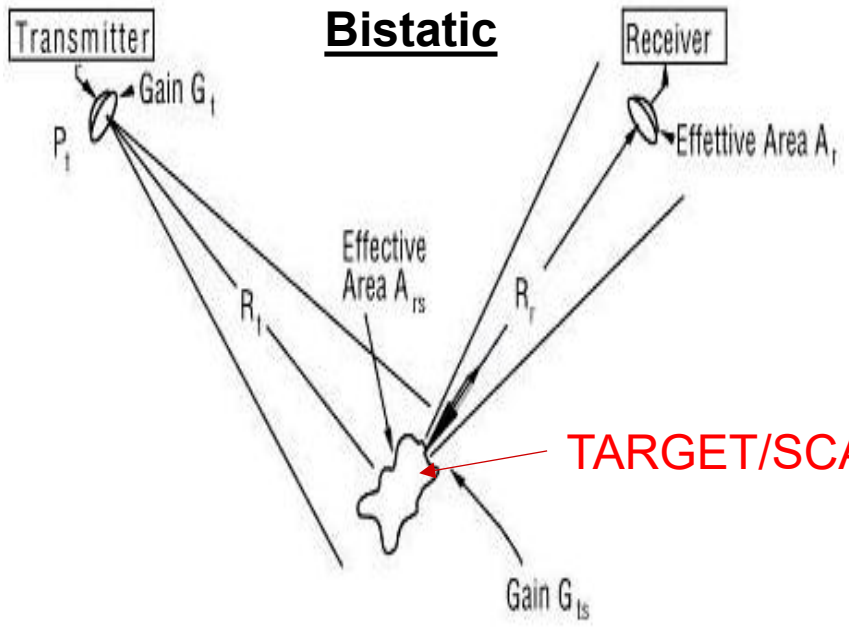
4. The **performance of the radar system** is usually theoretically assessed using the radar equation.

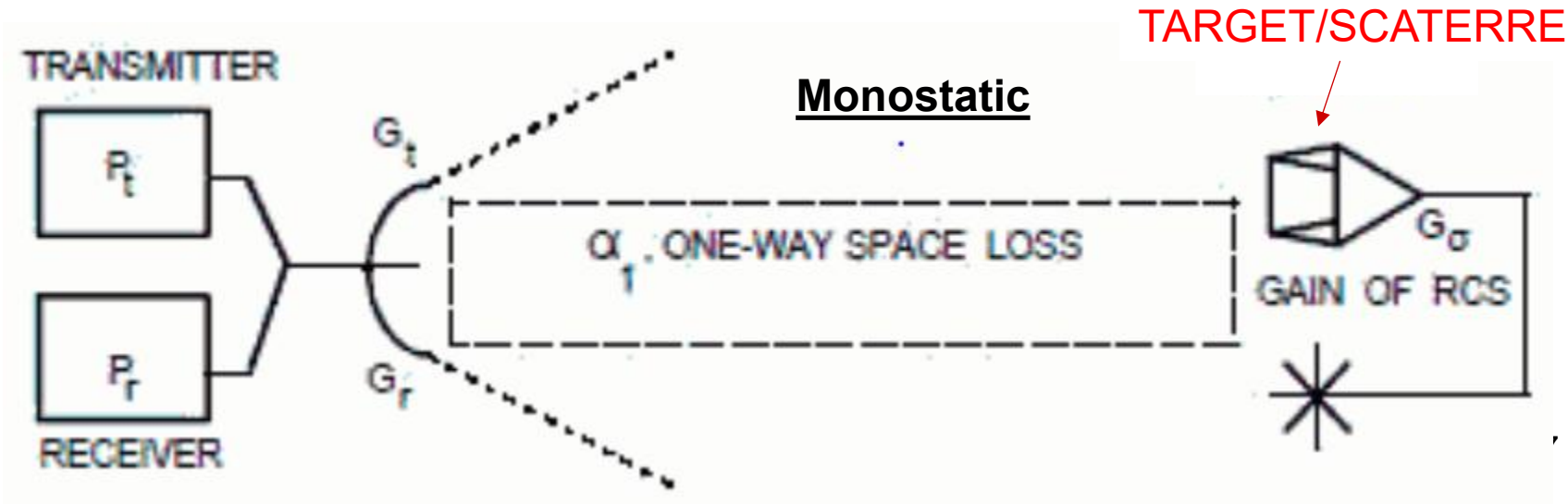
RADAR EQUATION: DEFINITION (2)

- **Radar Equation** represents the fundamental relation between the characteristics of the radar, the target, and the received signal.

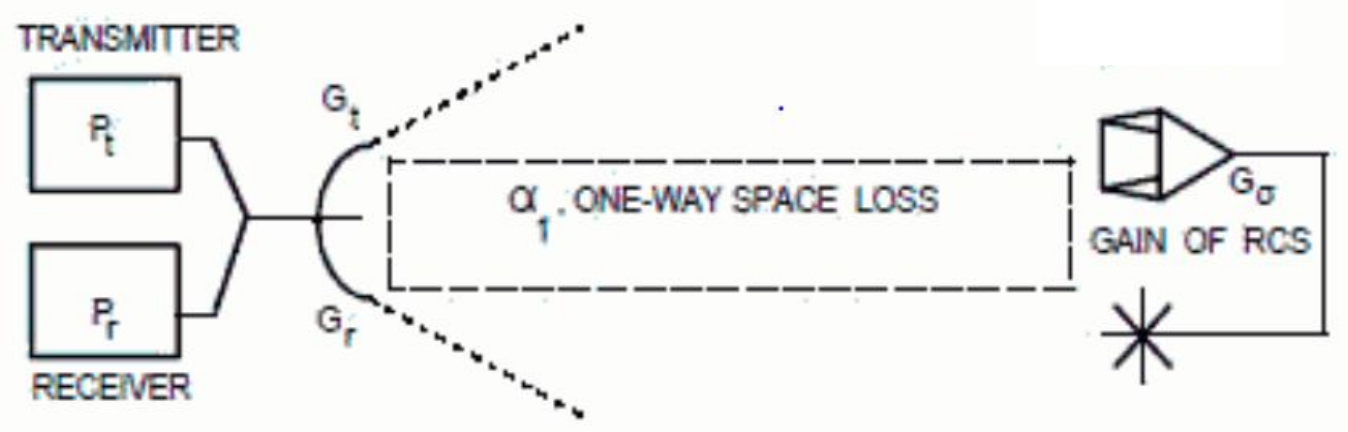


RADAR EQUATION – PHYSICAL CONCEPT

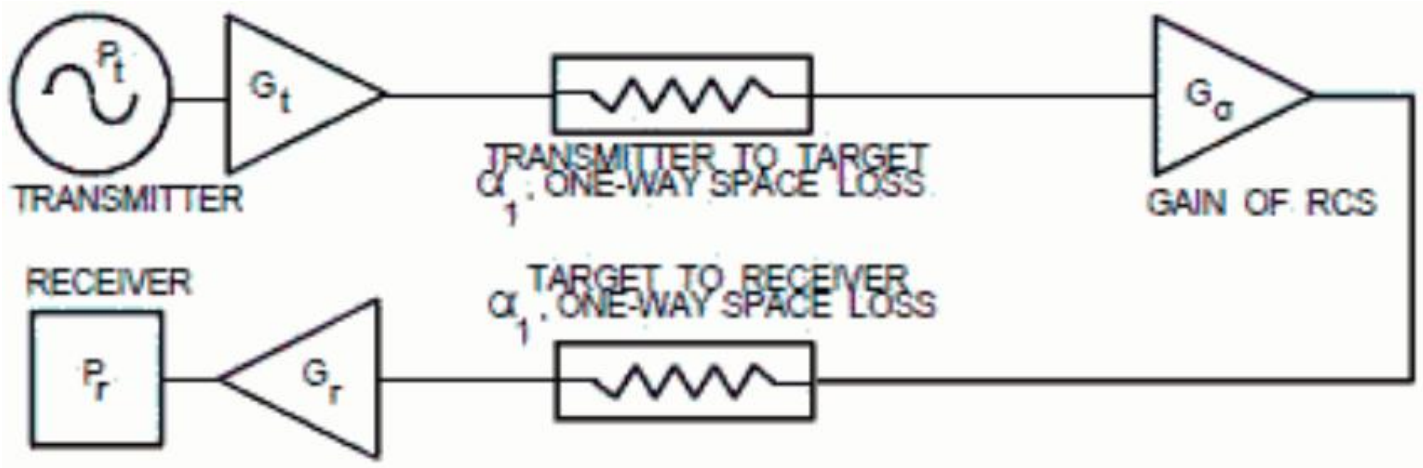




RADAR EQUIVALENT CIRCUITS



(a) Passive Radar (Uplink direction Only)

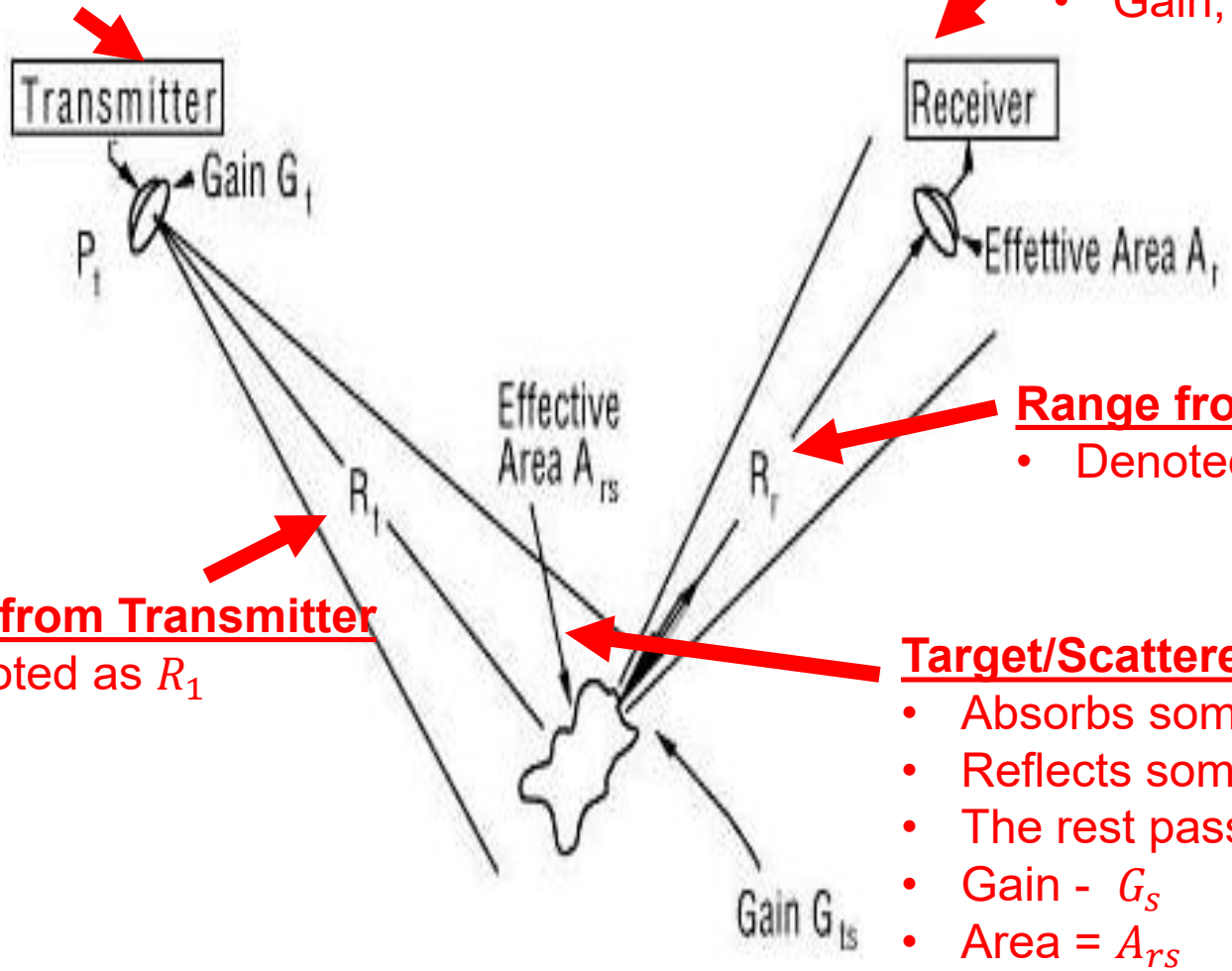


(b) Passive Radar (Uplink and Downlink)

RADAR EQUATION - PHYSICAL CONCEPT

Transmitter
- Transmits at power, P_t
- Gain, G_t

Receiver
- Effective area, A_r
- Gain, G_r



Range from Receiver
- Denoted as R_2

Range from Transmitter
- Denoted as R_1

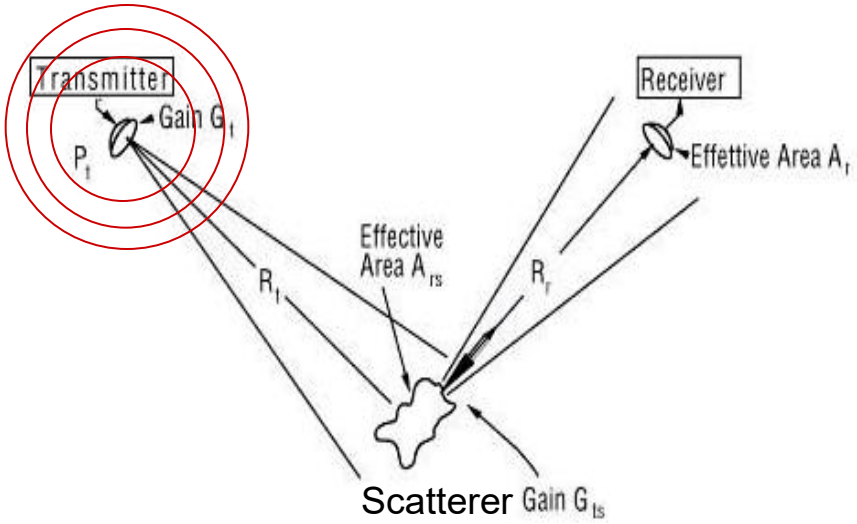
Target/Scatterer
- Absorbs some of the energy
- Reflects some of the energy
- The rest passes through
- Gain - G_s
- Area = A_{rs}

RADAR EQUATION: DERIVATION (1)

1. If high-frequency energy is emitted by an isotropic transmitter, the energy propagate uniformly in all directions.
2. The same amount of energy spreads out on an incremental basis on spherical surface with an incremented spherical radius.
3. That means: the power density on the surface of a sphere is inversely proportional to the square of the radius of the sphere and can be represented as:

$$S_s = \frac{P_t G_t}{4\pi R_1^2}$$

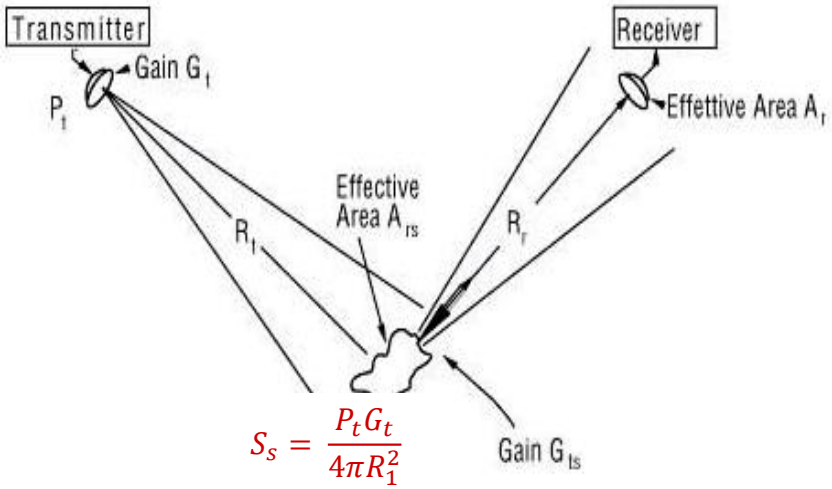
where S_s is the power density at the scatterer.



RADAR EQUATION: DERIVATION (2)

4. The scatterer intercepts a portion of the power and reflects (reradiates) it in the direction of the radar.

5. The amount of power reflected toward the radar receiver is determined by the Radar Cross Section (RCS) of the target. RCS is a characteristic of the target that represents its size as seen by the radar and has the dimensions of area.



6. We therefore multiply the power spectral density at the scatterer with the effective receiving area of the scatterer, A_{rs} to get

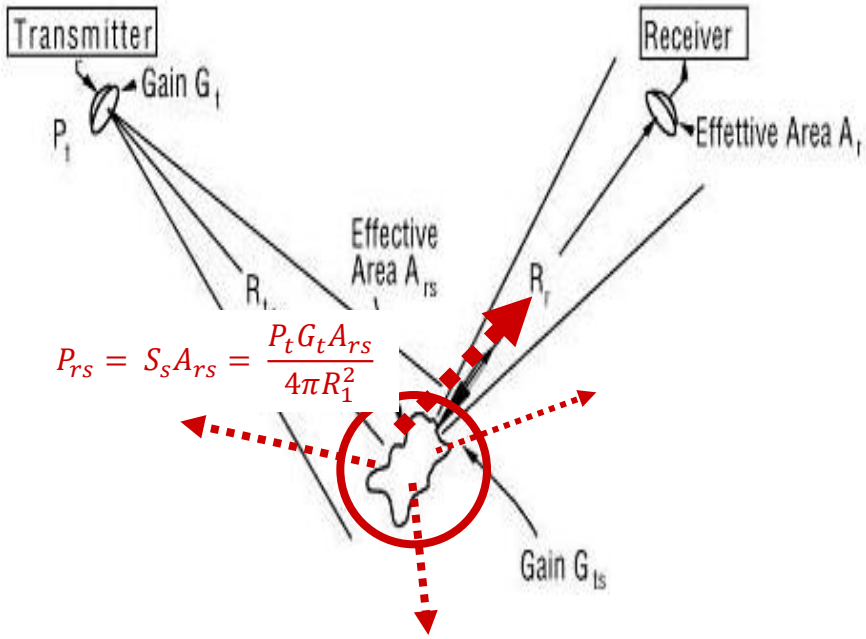
$$P_{rs} = S_s A_{rs} = \frac{P_t G_t A_{rs}}{4\pi R_1^2}$$

RADAR EQUATION: DERIVATION (3)

7. Some of the energy received by the scatter is absorbed while the rest is re-radiated in all directions.

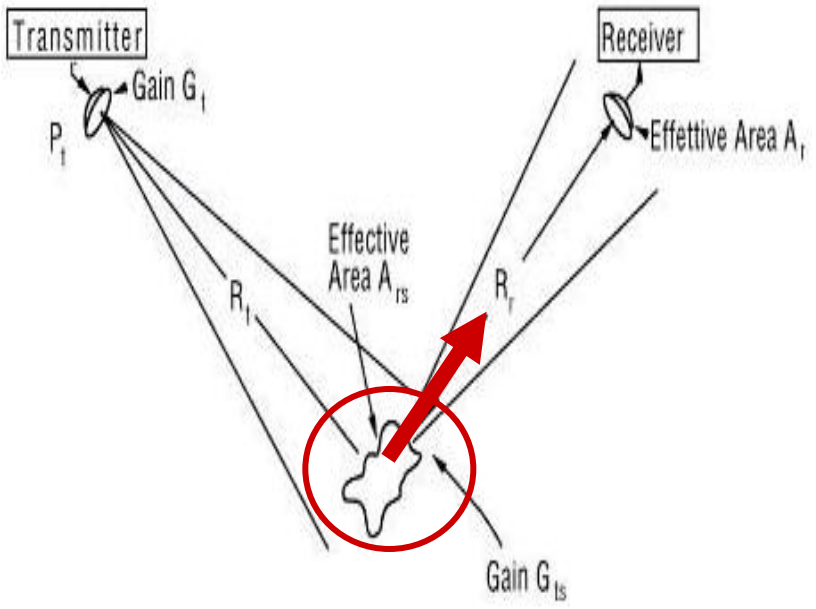
8. If the fraction of the absorbed energy is f_a then the re-radiated power, P_{ts} is given by

$$P_{ts} = \frac{P_t G_t A_{rs}(1 - f_a)}{4\pi R_1^2}$$



RADAR EQUATION: DERIVATION (4)

9. The conduction and displacement currents that flow in the scatterer result in re-radiation that has a pattern (like an antenna pattern).
10. The effective receiving area of the scatterer is a function of its orientation relative to the incoming beam, so that A_{rs} applies only for the direction of the incoming beam.
11. The re-radiation pattern may not be the same as the pattern of $\mathbf{A_{rs}}$, and the gain in the direction of the receiver is the relevant value in the re-radiation pattern.



12. P_{ts} is the total reradiated power, G_{ts} is the gain of the scatterer in the direction of the receiver and S_r is called the spreading factor for the re-radiation.

RADAR EQUATION: DERIVATION (5)

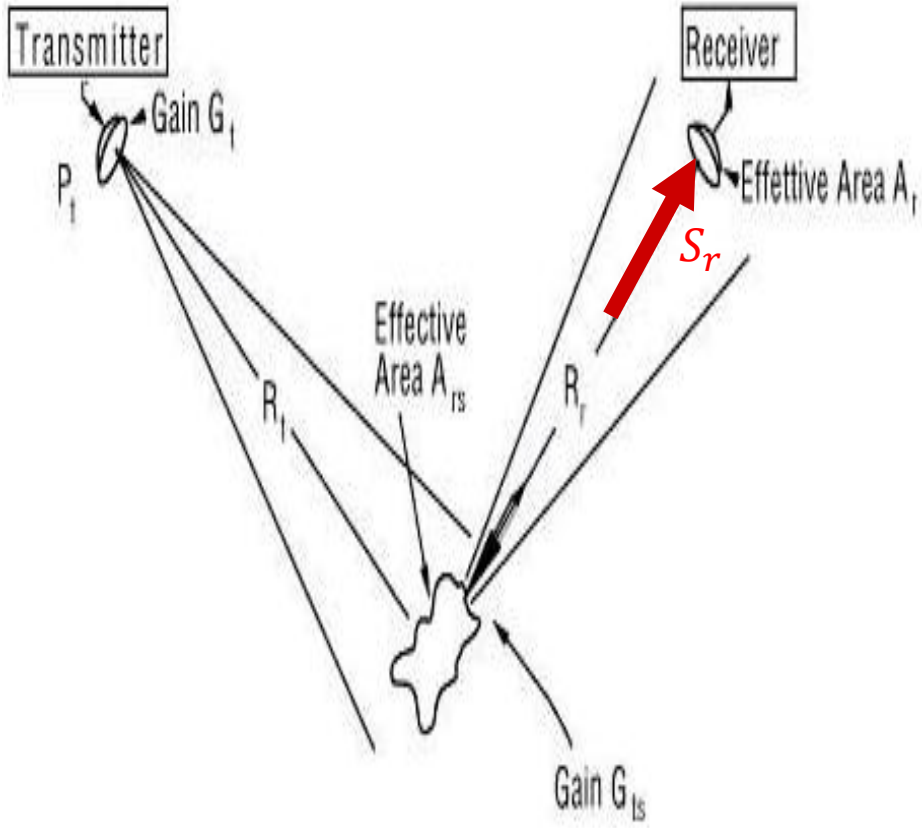
13. The power entering the receiver Radar is

where $$P_r = S_r A_r$$

- A_r is the effective aperture of the receiving antenna, not its actual area.
- S_r is the power density of the echo at the receiver.

13. Not only is A_r function of direction, but it is also a function of the load impedance the receiver provides to the antenna.

14. For example, P_r =0 if the load were a short circuit or an open circuit.



RADAR EQUATION: DERIVATION (6)

18. Combining equations discussed in 1-17 above yields:

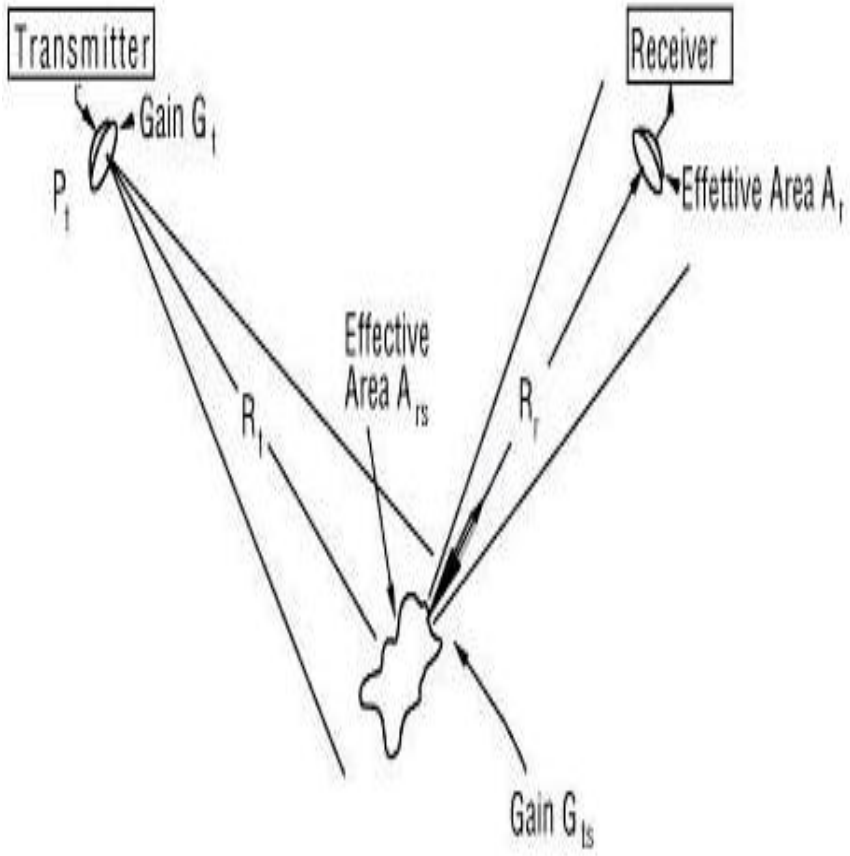
$$P_r = \frac{P_t G_t A_{rs}(1-f_a)}{4\pi R_1^2} \times \frac{G_r A_r}{4\pi R_2^2}$$

$$= \frac{P_t G_t A_r G_r}{(4\pi)^2 R_1^2 R_2^2} [A_{rs}(1-f_a)G_{ts}]$$

18. The factors associated with the scatterer are combined in the square brackets.

19. These factors are difficult to measure individually, and their relative contributions are not of interest to one wishing to know the size of the received radar signal. Hence they are normally combined into one factor, the radar scattering cross section:

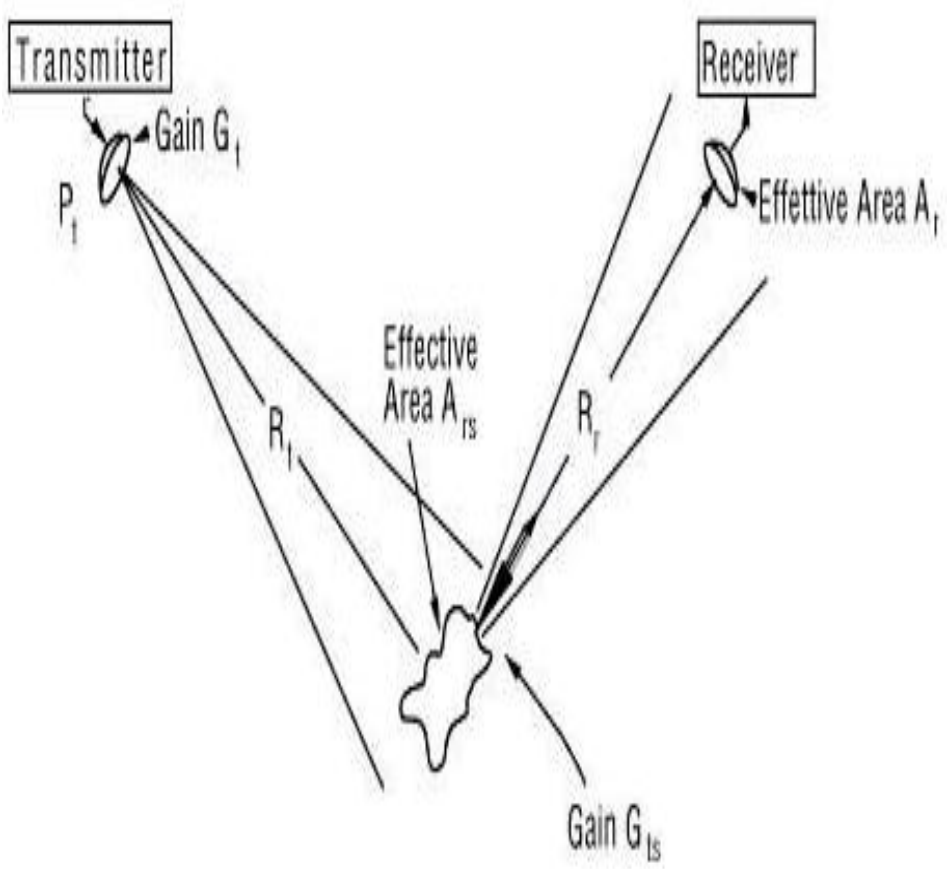
$$\sigma = A_{rs}(1-f_a)G_{ts}$$



RADAR EQUATION: DERIVATION (7)

- The final form of the radar equation is:

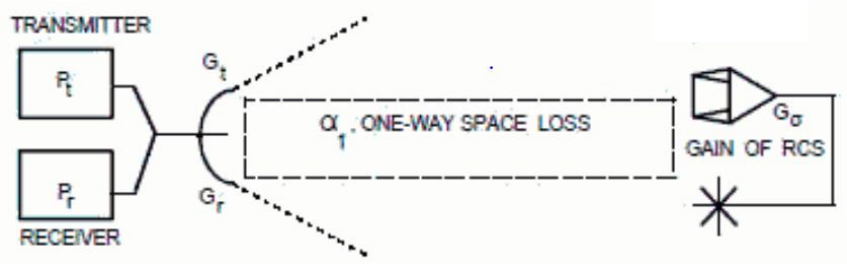
$$P_r = \frac{P_t G_t A_r G_r}{(4\pi)^2 R_1^2 R_2^2}\sigma$$



RADAR EQUATION: MONOSTATIC RADAR

- In monostatic radar,

$$R_1 = R_2 = R$$
$$G_t = G_r = G$$
$$A_r = A_t = A$$



- The radar equation therefore becomes:

$$P_r = \frac{P_t G_t A_r G_r}{(4\pi)^2 R_1^2 R_2^2} = \frac{P_t G^2 A_r \sigma}{(4\pi)^2 R^4}$$

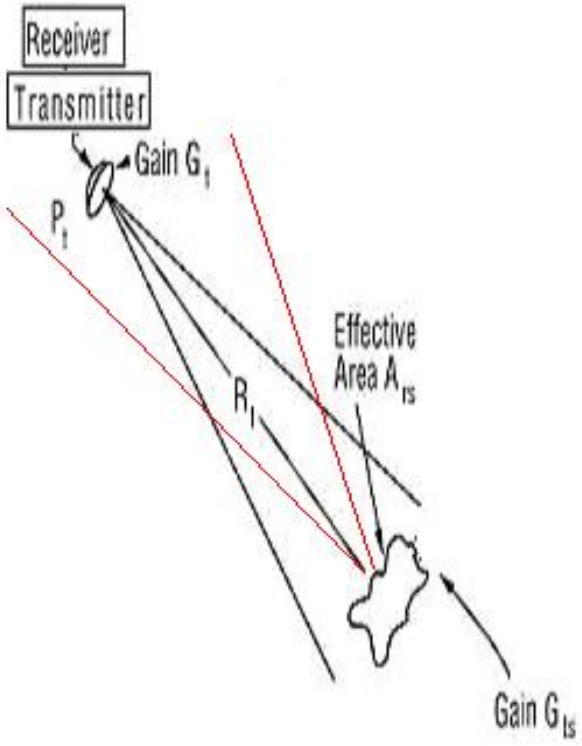
RADAR EQUATION: DERIVATION (9)

1. The effective area (effective aperture) of an antenna is related to its gain by

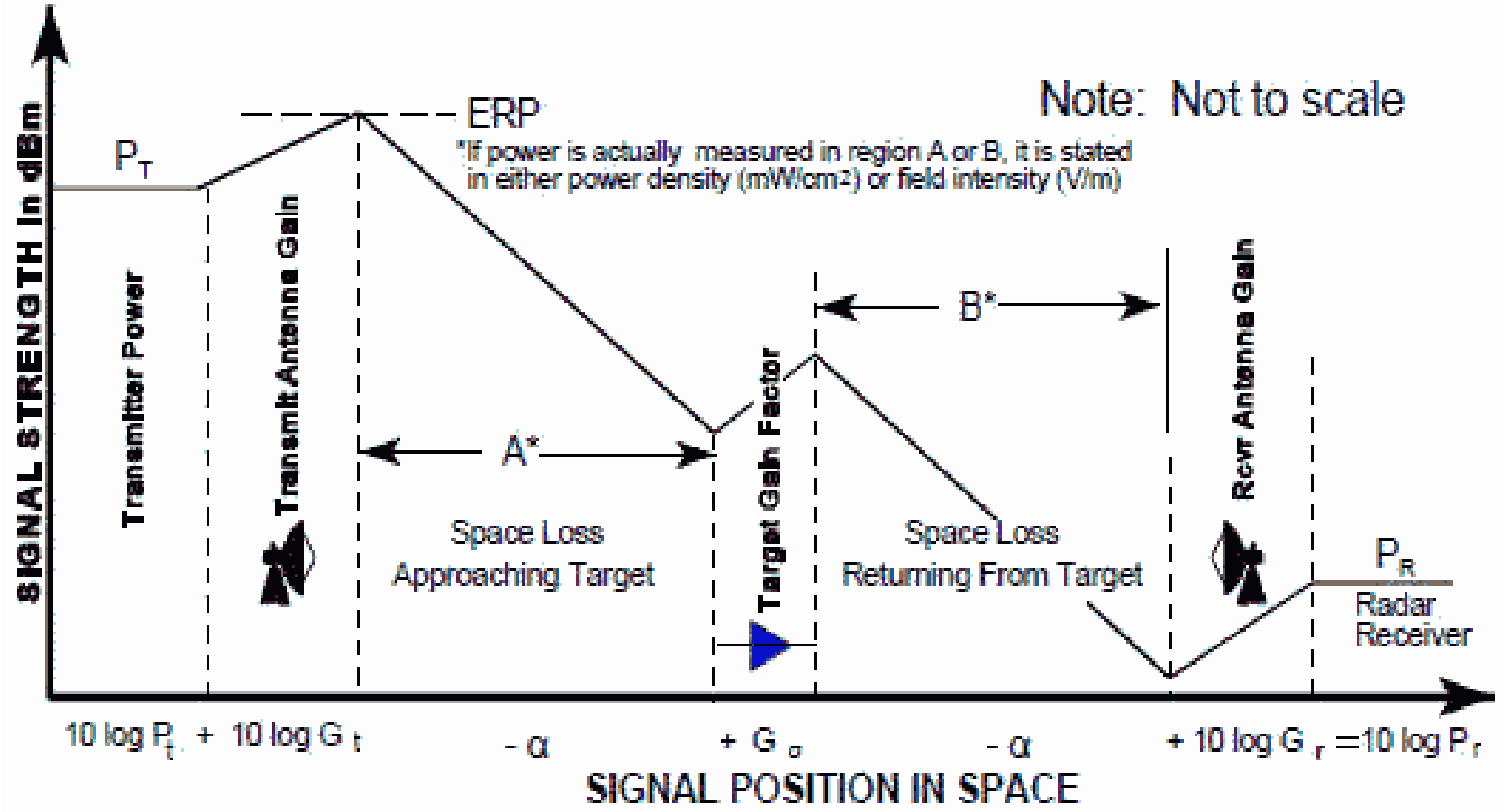
$$A = \frac{\lambda^2}{4\pi}$$

2. Therefore, the radar equation can be written as:

$$P_r = \frac{P_t G_t A_r G_r}{(4\pi)^2 R_1^2 R_2^2} = \frac{P_t G^2 A_r \sigma}{(4\pi)^2 R^4} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$



VISUALIZATION OF THE RADAR EQUATION



ONLINE RESOURCES

1. Radar Equation derivation:

- Radar Equation Derivation - Electronics and Communication Study Materials

2. Online Radar Range Calculator:

- Radar Maximum Range Calculator (pasternack.com)
- Radar Range Calculator - everything RF

LIMITATIONS OF THE RADAR EQUATION IN RANGE DETERMINATION

1. There are fluctuations and uncertainties in target radar cross section which it does not take into account.
2. The minimum detectable signal is affected by the receiver noise.
3. There are losses in the radar TRX system.
4. Propagation effects of the earth and the atmosphere are not taken into account.